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Compton-like interaction of massive neutrinos with virtual photons

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Abstract

The amplitude of a Compton-like process $\nu_i \gamma^* \to \nu_j \gamma^*$ with virtual photons is calculated in the standard GWS theory with lepton mixing. The contribution of this process to the high energy neutrino scattering on the nucleus with single photon radiation $\nu N \to \nu N \gamma$ is discussed. The bremsspectrum and the total cross-section are estimated in the leading log approximation.

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Electromagnetic properties of neutrinos as higher-order effects of weak interactions are of considerable interest and have a long history of discussions. Their possible manifestations are the Compton-like photon-neutrino scattering $\nu\gamma \to \nu\gamma$ and crossed reactions. However, as was shown by Gell-Mann [1] in the local four-fermion $(V-A)$ theory of weak interaction, the amplitude of such a process becomes zero when massless neutrinos and real photons are considered. It can be understood if $\nu\bar{\nu} \to \gamma\gamma$ process is analyzed in the center-of-mass system. In this case the total angular momentum of the neutrino-antineutrino pair can be equal only to a unit while the wave-function of the two on-shell photon system does not have such a state [2]. The amplitude becomes nonzero if any of the Gell-Mann theorem conditions is broken. It may be non-locality of neutrino interaction, non-zero neutrino mass or off-shell photons. The initial calculation of $\gamma^*\gamma \to \nu\bar{\nu}$ amplitude with one off-shell photon and $m_\nu = 0$ was made by Rosenberg [3] also in the frame of four-fermion theory. In that paper the $\gamma \to \nu\bar{\nu}$ process in the coulombian field of the nucleus was considered as an additional mechanism of the energy loss by stars. The leading local term for $\nu\gamma^* \to \nu\gamma^*$ with two virtual photons and $m_\nu = 0$ was found [4] in the gauge theory of weak interaction. An exact expression for the amplitude of the process $\gamma\gamma \to \nu\bar{\nu}$ with massive neutrinos and on-shell photons was obtained for the first time in ref. [5] (see also ref. [6]) to estimate the star energy loss due to this reaction.

Here we find the most general expression for the amplitude of $\nu_i\gamma^* \to \nu_j\gamma^*$ process (in general $i \neq j$) in the standard model of electroweak interaction embracing all the cases of real and virtual photons, massive and massless neutrinos and taking into account the mixing in the lepton sector. We consider the approximation $(pq_{1,2}) \ll m_W^2$ where $q_{1,2}$ and p are the four-momenta of photons and neutrino, respectively. The diagrams that give the main contribution to the process in this limit are shown in figs. 1 and 2. Then the amplitude can be written in the form

$$
\begin{aligned}
M &= \frac{\alpha}{\pi}\,\frac{G_F}{\sqrt{2}}\, j_\rho^{(\nu)} \left(\sum_l V_{il}^* V_{jl} R_\rho^{(l)} \;+\; \delta_{ij} \sum_f T_{3f} Q_f^2 R_\rho^{(f)} \right), && (1)\\
R_\rho^{(f)} &= 4i\, \{ \frac{1}{2} (F_1 \tilde{F}_2)(q_2 - q_1)_\rho \, A(m_f, q_1, q_2) \\
&- (\tilde{F}_2 F_1 q_1)_\rho \, B(m_f, q_1, q_2) + (\tilde{F}_1 F_2 q_2)_\rho \, B(m_f, q_2, q_1) \}, && (2)\\
(F_1 \tilde{F}_2) &= F_{1\mu\nu} \tilde{F}_{2\nu\mu}, \qquad (\tilde{F}_2 F_1 q)_\rho = \tilde{F}_{2\rho\mu} F_{1\mu\nu} q_{1\nu},
\end{aligned}
$$

$$
\begin{aligned}
F_{\mu\nu} &= q_\mu e_\nu - q_\nu e_\mu, \qquad \tilde{F}_{\mu\nu} = \frac{1}{2}\epsilon_{\mu\nu\alpha\beta}F_{\alpha\beta}, \qquad (3)\\
&A(m, q_1, q_2) = \int_0^1 xdx \int_0^{1-x} ydy\, \frac{1}{a},\\
&B(m, q_1, q_2) = \int_0^1 xdx \int_0^{1-x} dy\, \frac{1-x-y}{a}, \qquad (4)\\
a &= m^2 + 2(q_1 q_2)\, xy - q_1^2\, x(1-x) - q_2^2\, y(1-y). \qquad (5)
\end{aligned}
$$

Here $j_\rho^{(\nu)} = \bar{\nu}_j(p_2)\gamma_\rho(1-\gamma_5)\nu_i(p_1)$ is the neutrino $(V-A)$ current, e_μ is the photon polarization four-vector, V_{il} is the mixing matrix of Kobayashi-Maskawa type in the lepton sector. The first term in (1) is related to the W-contribution (fig.1) and needs the summation over all charged leptons. The second term in (1) comes from the Z-contribution (fig.2) where the sum runs over all charged fermions (both leptons and quarks). Here eQ is the electric charge of the fermion and T_{3f} is the third component of weak isospin. Let us note that the first term in eq.(2) is reduced to the divergence of the neutrino current and so it will be proportional to the neutrino mass. The obtained amplitude is explicitly gauge invariant since it is expressed in terms of electromagnetic tensors of the photons (3). In some particular cases it can be reduced to the well-known results of refs. [3–5]. For example, if we assume both photons to be virtual ($q_{1,2}^2 \neq 0$) and neutrino to be massless, the amplitude (1), (2) can be transformed to the one given by Cung and Yoshimura [4]. In our opinion, their amplitude contains the artificial dependence on the neutrino momenta. It is obvious, however, that in the considered approximation (that is in fact the local limit of weak interaction) the amplitude of the $\nu\gamma^* \to \nu\gamma^*$ process can manifestly depend on the photon momenta only.

Our general result (1) allows us also to find the first terms in the expansion for the amplitudes of the neutrino radiative decay $\nu_i \to \nu_j\gamma$ and non-radiative transition $\nu_i \to \nu_j$ in the external electromagnetic field of arbitrary configuration. For this purpose it is sufficient to replace the electromagnetic tensor of either one or both photons by the external field tensor.

We shall illustrate the result (1) considering the high energy neutrino scattering process in the Coulombian field of a nucleus with one photon radiation. Previously [3,5,6] only astrophysical effects of the process $\nu\gamma \to \nu\gamma$ were studied. Our aim is to examine the possibility of the detection of the process in the laboratory. Really, it could be observed as bremsstrahlung when the neutrino is scattered by a nucleus without its break-up,

$$\nu + nucleus \rightarrow \nu + \gamma + nucleus. \tag{6}$$

The reaction amplitude can be obtained from (1), (2) taking one of the photons (e.g. γ_2) to be real. In this case one has $F_{2\mu\nu}q_{2\nu} = 0$. To get over the technical difficulties we shall regard $m_\nu = 0$ and neglect the lepton mixing. Then the amplitude will be defined by the second term in (2). Inserting $(Ze/q_1^2)J_\mu$ instead of $e_{1\mu}$, where J_μ and Ze are the electromagnetic current and the charge of the nucleus, $q_{1\mu}$ and $e_{1\mu}$ are the momentum and the polarization vector of the virtual photon, one obtains

$$M = 4i\frac{Ze\alpha}{\pi}\frac{G_F}{\sqrt{2}}\epsilon_{\rho\mu\alpha\beta}j_\rho^{(\nu)}J_\mu q_{2\alpha}e_{2\beta}\left(B(m_\ell, q_1, q_2) + \sum_f T_{3f}Q_f^2 B(m_f, q_1, q_2)\right). \tag{7}$$

Here m_ℓ is the mass of the charged lepton which is the partner of the neutrino taking part in the reaction. Let us examine the case of small transmitted momenta when the nucleus is still nearly motionless. The momentum modulo $|\vec{q}_1|$ is restricted then by the value of q_m which can be estimated as the inverted nucleus radius $q_m \simeq 1/r \simeq 300$ MeV. One can easily see from eq.(4) that at high energies of the neutrino all the charged fermions contribute to the amplitude (7) except t-quark (we still presume $(pq_1) \ll m_W^2 < m_t^2$). In the leading log approximation we get the following expression for the spectrum of radiated photons:

$$d\sigma = \frac{\alpha}{54\pi}\left(\frac{Z\alpha}{\pi}\right)^2 \frac{G_F^2 q_m^2}{\pi}\frac{d\omega}{\omega}\left(1 - \frac{\omega}{E_\nu} + \frac{1}{2}\left(\frac{\omega}{E_\nu}\right)^2\right) \quad \ln^3\left(\frac{2\omega}{q_m}\right), \tag{8}$$

where ω is the photon energy, E_ν is the initial neutrino energy, q_m is the maximal momentum of the nucleus recoil. For the high energy neutrinos, within the above approximation the total cross-section of the process is

$$\sigma \simeq \left(\frac{\alpha}{2\pi}\right)^3 \frac{Z^2}{27}\frac{G_F^2 q_m^2}{\pi} \ln^4\left(\frac{2E_\nu}{q_m}\right). \tag{9}$$

For example, for a neutrino energy $E_\nu = 100$ GeV we have

$$\sigma \simeq Z^2 \cdot 1.6 \cdot 10^{-46}\ cm^2. \tag{10}$$

This small value of the cross-section makes it difficult to observe the bremsstrahlung in the neutrino scattering by the coulombian field of the nucleus. This is true even if one takes into account the distinctive signature of the reaction as the production of a high energy photon without any accompanying particles. It must be noted that the same signature in the neutrino reaction may correspond to the coherent production of photons by nucleons of the nucleus [7,8]. However, the process we consider has a narrower angular distribution of photons, $\theta < q_m/E_\nu$ instead of $\theta < \sqrt{q_m/E_\nu}$ [8, 9]. Moreover, it is necessary to distinguish in the neutrino experiment between the electromagnetic showers produced by photons and by recoiled electrons in the process $\nu e \to \nu e$ which has a cross-section 10^4 times larger than (10).

Nevertheless, we hope to overcome in the future the experimental difficulties we have pointed out. Then the process $\nu\gamma^* \to \nu\gamma$ we have discussed could be accessible to observation. This process (one-loop at the minimum) could be one of the few tests for the validity of higher-order perturbation theory in the standard model of electroweak interaction.

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